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Technical Comments

Comment on "Equation Decoupling— A New Approach to the Aerodynamic Identification of Unstable Aircraft"

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FOR the following linear system, with measurement matrix equal to I

$$\dot{X}(t) = F(\pi)X(t) + G(\pi)u(t) \tag{1}$$

$$y(t) = X(t) + n(t) \tag{2a}$$

An equation decoupling technique is presented in Ref. 1. By introduction of two $(n \times n)$ matrices KO and KOI, whose elements under the following restrictions:

$$KO(i, j) = 0, 1 \qquad i, j = 1, 2, \ldots, n \tag{2b}$$

$$KO(i, i) = 1 \qquad i = 1, 2, \ldots, n \tag{2c}$$

If

$$KO(i, j) = KO(i, k) = 1$$

then

$$KO(j, k) = 1 \qquad i, j, k = 1, \ldots, n \tag{2d}$$

$$KOI(i, j) = 1 - KO(i, j) \qquad i, j = 1, \ldots, n \tag{2e}$$

the linear system [Eq. (1)] is changed into

$$\dot{X}(t) = F(\pi)[KOX(t) + KOIy(t)] + G(\pi)u(t) \tag{3}$$

Reference 1 points out that for the completely decoupled case, $KO = I$, and for the completely coupled case, $KOI = 0$. The equation decoupling technique indeed lead to some advantages, but there are two problems that have not been clearly explained.

In Ref. 1, KO and KOI do not represent matrices, $F(\pi)\cdot KO$ and $F(\pi)\cdot KOI$ also do not represent matrix multiplication. They meet the following relations:

If

$$\begin{aligned} KO(i, j) &= 1 \qquad Fij(\pi)KO(i, j) = Fij(\pi) \\ KO(i, j) &= 0 \qquad Fij(\pi)KO(i, j) = 0 \end{aligned} \tag{4}$$

$$\begin{aligned} KOI(i, j) &= 1 \qquad Fij(\pi)KOI(i, j) = Fij(\pi) \\ KOI(i, j) &= 0 \qquad Fij(\pi)KOI(i, j) = 0 \end{aligned} \tag{5}$$

when only the ith equation is decoupled:

$$\begin{aligned} &KO(i, i) = 1 \\ &KO(i, j) = KO(j, i) = 0 \\ &j = 1, 2, \ldots, i-1, i+1, \ldots, n \end{aligned} \tag{6}$$

Because of the introduction of the measured variables $y(t)$ into the system by way of the matrix KOI, the measured noise $n(t)$ is introduced into state equations [Eq. (3)], and makes the system model stochastic. The process noise introduced by KOI is now cross correlated. For Eq. (4) of Ref. 1, the process noise is

$$\begin{bmatrix} 0 & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & 0 & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & 0 & \pi_{34} \\ \pi_{41} & \pi_{42} & \pi_{43} & 0 \end{bmatrix} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \tag{7}$$

where n_i ($i = 1, 2, 3, 4$) is measurement noise. If it is assumed that no correlation between the measurement noise components exists, then the process noise covariance matrix can be expressed as

$$\begin{aligned} q_{11} &= \pi_{12}^2\sigma_2^2 + \pi_{13}^2\sigma_3^2 + \pi_{14}^2\sigma_4^2 \\ q_{22} &= \pi_{21}^2\sigma_1^2 + \pi_{23}^2\sigma_3^2 + \pi_{24}^2\sigma_4^2 \\ q_{33} &= \pi_{31}^2\sigma_1^2 + \pi_{32}^2\sigma_2^2 + \pi_{34}^2\sigma_4^2 \\ q_{44} &= \pi_{41}^2\sigma_1^2 + \pi_{42}^2\sigma_2^2 + \pi_{43}^2\sigma_3^2 \end{aligned} \tag{8}$$

where σ_j^2 is the variance of the measurement noise n_j for $j = 1, 2, 3, 4$.

The process noise will effect the resulting estimates. Accurate results cannot be obtained unless the measurement noise n_i is very small and can be neglected. Real flight test data often contains measurement noise that cannot be neglected.

The authors give the following suggestion: First, the equation decoupling technique is used to get Eq. (3), then, if the measurement noise cannot be neglected, a filter error method[1,2] is applied to identify the parameters. Such a procedure makes use of the advantage of the reduction of the complexity of the sensitivity functions. Although this procedure is more time consuming and complex than Ref. 1, accurate results can be obtained.

References

[1]Preissler, H., and Schäufele, H., "Equation Decoupling — A New Approach to the Aerodynamic Identification of Unstable Aircraft," *Journal of Aircraft*, Vol. 28, No. 2, 1991, pp. 146–150.

[2]Jategaonkar, R. V., and Plaetschke, E., "Identification of Moderately Nonlinear Mechanics Systems with Additive Process and Measurement Noise," *Journal of Guidance Control and Dynamics*, Vol. 13, No. 2, 1990, pp. 277–285.

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